EVALUATING DYNAMIC STRESSES OF A PIPELINE

by

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INTRODUCTION

Piping vibration is as much of a concern to utility owners and operators as to federal regulatory bodies. Many programs have been developed to assure reliability and plant safety with respect to vibration while minimizing cost and delay during plant start-up. The acceptance of a piping system vibration is determined by the maximum vibratory stress in the pipe. This could be determined by either visual observation or by more complicated instrumentation measurement and analysis techniques as the situation may require. In either case, the dynamic stress should not exceed an allowable level defined from permissible alternating stress values given by the ASME Code for a given number of cycles. Since direct dynamic stress measurement is a complicated process, vibration is mainly monitored by using portable instruments to perform frequency and amplitude measurements. The following approach presents the fast and reliable way to evaluate the harmonic dynamic stresses of a simply supported pipeline from the data collected on the field. Furthermore, with the versatility of the personnel computer the piping stress software's user could obtain fairly easily the dynamic stress without having a solid background of the dynamic vibration. This present approach also offers a basic understanding to solve quickly vibration problem when and where the computer software is not accessible.

BASIC EQUATIONS

Free vibration occurs when a system is displaced from its static position and left free to oscillate. Under free vibration the system oscillates at its *natural frequencies*. The natural frequencies are dynamic characteristics of the system specified by its stiffness and inertia properties. Natural frequencies are calculated with *modal analysis*. Forced vibrations are classified into *periodic and non-periodic*. In a periodic vibration, the response repeats itself at a regular time interval, called *period* T. Harmonic excitation is a sub-class of periodic vibration and is referred hereafter as an analytical approach for the present investigation.

Consider an undamped single degree of freedom SDOF system that is subjected to a harmonic force P(t) with amplitude P_o and circular frequency, ω_f

(Fig. 1). The equation of motion is given by

$$M\ddot{y} + ky = P_o \sin\omega_f t \qquad (1)$$

The solution of Eq. 1 is

$$y(t) = A\cos\omega t + B\sin\omega t + \frac{P_o}{k}\frac{1}{1 - r^2}\sin\omega_f t \qquad (2)$$

where r is defined as the ratio of the circular frequency of the externally applied load to the natural circular frequency of the system, that is

$$r = \frac{\omega_f}{\omega} = \frac{f_f}{f} \tag{3}$$

The solution given by equation 2 is the superposition of the free vibration problem and the effect of the exciting force exposed by the last term of Eq. 2 which involves only the frequency of the harmonic load. For frequency response analysis or steady state harmonic analysis, only the steady state response is considered and Eq. 2 becomes

$$y(t) = \frac{P_o}{k} \frac{1}{1 - r^2} \sin \omega_f t \tag{4}$$

The solution for maximum displacement from an unphased harmonic analysis is then

$$\delta_{dyn} = \frac{P_o}{k} \frac{1}{1 - r^2} \tag{5}$$

Let $\frac{P_o}{k} = \delta_{static}$

$$\delta_{dyn} = \frac{\delta_{static}}{1 - r^2} \tag{6}$$

where k is the piping structural stiffness and δ_{static} is the static deflection of the system.

For the simple hinged supports pipe of mass intensity m, we introduce the boundary conditions :

(B.C.1) $y = 0$ at $x = 0,L$

(B.C.2) $\frac{d^2y}{dx^2} = 0$ at $x = 0,L$

and the natural frequencies(1) are :

$$f_n = \frac{n^2\pi}{2L^2}\sqrt{\frac{EI}{m}} \tag{7}$$

where—

- E = Young’s Modulus (psi)
- I = Moment of inertia of pipe cross section (in^4)
- m = Mass intensity ($lbs\text{-}sec^2/in^2$)
- L = Length of pipe (in)
- n = 1, 2, 3, …

Statically, when the pipe is loaded with a force F at mid-span, the beam deflection equations (as shown in Fig. 2) are well known and written as follows[2] :

At load

$$\Delta = \frac{PL^3}{48EI} \tag{8}$$

When x < L/2

$$\Delta = \frac{Px}{48EI}\left(3L^2 - 4x^2\right) \tag{9}$$

The deflection is symmetrical both sides of the concentrated load position and the dynamic deflection could be evaluated using equations 6, 8, and 9.

With a typical dynamic deflection profile as shown in Fig. 3, in order to determine dynamic stresses one needs only apply the well-known relationships

$$M = -EI\,\frac{d^2y}{dx^2} \tag{10}$$

$$\sigma = \frac{MD}{2I} \tag{11}$$

for bending moment and bending stresses.

The second derivative of Eq. 10 could be solved using finite differences technique. Since we are interested in the maximum stress evaluation, the second derivative at mid-span are calculated as follows :

$$\ddot{y}_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \tag{12}$$

where

- h = Equi-spaced argument
- y_i = Dynamic deflection at load (x = L/2)
- y_{i+1} & y_{i-1} = Dynamic deflection at (x = L/2 ± h)

The dynamic peak stress is then, using Eq. 8, 9 and 12

$$\sigma_{dyn} = \frac{PD}{48Ih^2(1 - r^2)}\left(3L^2x - 4x^3 - L^3\right) \tag{13}$$

and

$$\left.\frac{\delta}{\sigma}\right|_{dyn} = \frac{L^3h^2}{ED(3L^2x - 4x^3 - L^3)} \tag{14}$$

Observing the relation 14, it is interesting to note that the ratio of dynamic deflection to dynamic stress is constant for a pipe size in consideration, and it is independent of the pipe wall thickness as well as the harmonic excitation frequency and load.

It should be noted that the same technique could be used to develop a general case where the concentrated load is at any point between the two (2) supports (Fig. 4). The beam static deflection equations are as follows :

At load

$$\Delta = \frac{Pa^2b^2}{3EIL} \tag{15}$$

When x<a

$$\Delta = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right) \tag{16}$$

When x>a

$$\Delta = \frac{Paz}{6EIL}\left(L^2 - a^2 - z^2\right) \tag{17}$$

The moment due to a harmonic force excitation is then :

$$M = EI\left[\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}\right] \tag{18}$$

where the term in the bracket could be measured directly on the site or calculated using Eq. 6 combined with Eq. 16, 15 and 17 respectively.

<u>CASE STUDY</u>

In the following case study, we will show the application of this development and compare the results with the one produced by a piping stress software with dynamic analysis feature.

The piping model shown in Fig. 5 has the following characteristics :

Total Length	240 in
Outside Diameter	4.5 in
Wall Thickness	0.337 in
Modulus of Elasticity	30E6 psi
Mass Density	0.2825 lb / in^3
Number of Elements	22
Boundary Conditions	Simply supported

A harmonic force of 200 lbs is applied at its mid-span with an excitation frequency of 5 Hz. In reality these loads could be for example a in-line pump with its own dead weight, sits on the piping structure and runs at a certain rpm. Evaluate the maximum dynamic stress and peak displacement.

In-situ measurement and evaluation :

- Use vibration probe and measure peak displacement at the center of the pipe span to get y_i and at an adjacent location of distance h from the center for y_{i-1} or y_{i+1}

  The distance h is usually selected to be 4 to 5 times the pipe size in such a way that the displacement readings are not too close between them.

- Use Eq. 12 to calculate $\ddot{y}_i$
- Use Eq. 10 to calculate the moment $M = EI\ddot{y}_i$
- Use Eq. 11 to evaluate the peak dynamic stress.

Analytical calculation :

1 Fundamental frequency $f_n = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m}}$ (Eq 7)

Moment of inertia :

$$I=0.049087(D^4 - d^4)$$
$$=0.049087(4.5^4 - 3.826^4)$$
$$=9.61 \text{ in}^4$$

Mass intensity : m=(A / g)

$$= \frac{0.2825}{32.174}\ \frac{\pi}{4}\ \frac{D^2 - d^4}{12}$$
$$=0.003225 \text{ lb-sec}^2 / \text{in}^2$$

Fundamental frequency :

$$f_n = \frac{\pi}{2(240)^2}\sqrt{\frac{(30E6)(9.61)}{0.003225}} =8.154 \text{ Hz}$$

2 Frequency ratio :

$$r = \frac{f_f}{f} = \frac{5}{8.154} = 0.6132 =5 / 8.154 = 0.6132 \quad r^2 = 0.376$$

3 Evaluation of dynamic stress (eq 13).

Using the equi-distance h of 12 in :

$$\sigma_{dyn} = \frac{(200)(4.5)}{48(9.61)(12)^2(1 - 0.376)}\left[3(240)^2(108) - 4(108)^3 - 240^3\right]$$

$$= \quad 4352 \text{ psi}$$

4 Dynamic peak deflection (eq 14):

$$\left.\frac{\delta}{\sigma}\right|_{dyn} = \frac{(240)^3(12)^2}{30E6(4.5)\left[3(240)^2(108) - 4(108)^3 - 240^3\right]} = 7.356E\text{-}5 \text{ in}^3 / \text{lb}$$

$$\delta_{dyn} = \sigma_{dyn}(7.356E\text{-}5) = 0.3201 \text{ in}$$

COMPUTER SIMULATION :

A harmonic dynamic analysis of the above system has been performed using a piping stress analysis computer code. The harmonic load of 200 lbs with a frequency excitation of 5 Hz was applied at Node 12 (mid-span). A mode shape analysis has been also performed to evaluate the first three (3) natural frequencies which are 8.15, 32.5 and 72.75 Hz. The computer output is tabulated in the same Table 1 for comparison purpose. As we can see, the displacement as well as dynamic stress results are nearly identical in both calculations.

- Stress :

$$\varepsilon = \left|\frac{4352\text{-}4190}{4352}\right| \text{ x } 100 = 3.72\%$$

- Displacement :

$$\varepsilon = \left|\frac{0.3201\text{-}0.3195}{0.3201}\right| \text{ x } 100 = 0.18\%$$

REFERENCES

1. Biggs, J.M., "Introduction to Structural Dynamics", McGraw Hill Publishing Company.

2. Handbook of Steel Construction, CISC, Fifth Edition 1991.

TABLE 1 : RATIO OF $\left.\frac{\delta}{\sigma}\right|_{dyn}$
FOR VARIOUS LOADS FOR THE SPAN L = 240 IN.

SYSTEM IN CONSIDERATION				COMPUTER SIMULATION			Eq. 14
Pipe O.D. (in.)	Pipe wall (in.)	Force (lb)	Frequency (Hz)	Deflection (in.)	Stress (psi)	$\left.\frac{\delta}{\sigma}\right\|_{dyn}$	$\left.\frac{\delta}{\sigma}\right\|_{dyn}$
4.5	0.337	200	5	0.3195	4190	7.62E-5	7.35E-5
4.5	0.740	1000	5	1.0775	13948	7.72E-5	7.35E-5
4.5	0.337	400	3	0.4624	6336	7.3E-5	7.35E-5